Ma PC1A CT1

(1) $f(x) = 2\sqrt{x} - 3\ln(x+2)$

(a) $D_f = [0, +\infty[\cap]-2, +\infty[= [0, +\infty[= \mathbb{R}_+$

(b) $\lim\limits_{x \to 0^+} f(x) \underset{\text{par continuité}}{=} f(0) = -3\ln(2)$

$\lim\limits_{x \to +\infty} f(x) = [+\infty - \infty]$ F.I.

$= \lim\limits_{x \to +\infty} \sqrt{x}\left(2 - \dfrac{3\ln(x+2)}{\sqrt{x}}\right) = +\infty$

car : $\lim\limits_{x \to +\infty} \dfrac{3\ln(x+2)}{\sqrt{x}} \underset{\text{Hôpital}}{=} \lim 6 \dfrac{\sqrt{x}}{x+2} = 0$

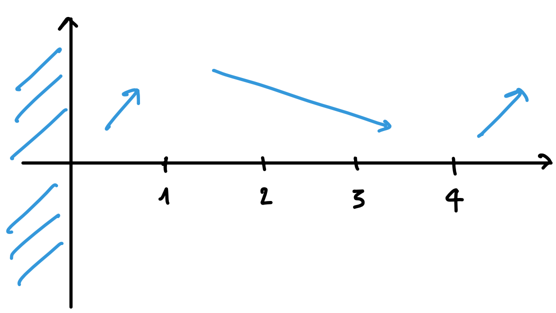
(c) $f'(x) = \dfrac{1}{\sqrt{x}} - 3\dfrac{1}{x+2}$

$D_{f'} =]0, +\infty[$

(d) $f'(x) = 0 \iff x + 2 = 3\sqrt{x} \iff x^2 - 5x + 4 = 0$

$x_1 = 1$, $x_2 = 4$

(e) $f'(x) > 0 \iff x + 2 > 3\sqrt{x} \iff x \in]0,1[\cup]4, +\infty[$



maximum local en $x = 1$
minimum local en $x = 4$

(2)

(a) $\displaystyle\int \frac{1}{(x-1)^2}\,dx = -\frac{1}{x-1} + c$

(b) $\displaystyle\int \frac{1}{x^2-1}\,dx = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + c$

(c) $\displaystyle\int \frac{1}{x^2+1}\,dx = \arctan(x) + c$

(3)

(a) $\displaystyle\int_0^{\pi} \frac{\sin t}{1+\cos^2 t}\,dt = \int_1^{-1} \frac{-dy}{1+y^2} = \arctan y \Big|_{-1}^{1} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$\cos t = y$ $\qquad \cos 0 = 1$

$-\sin t\,dt = dy$ $\qquad \cos\pi = -1$

(b) $\displaystyle\int_0^{\pi/2} e^x \cos(x)\,dx = e^x \cos x + \int e^x \sin x\,dx =$

$\displaystyle= e^x \cos x + e^x \sin x - \int e^x \cos x\,dx$

$\displaystyle\Rightarrow \int_0^{\pi/2} e^x \cos(x)\,dx = \frac{1}{2}\Big[e^x \cos x + e^x \sin x\Big]_0^{\pi/2} =$

$\displaystyle= \frac{1}{2} e^{\pi/2} - \frac{1}{2}$

(4) $g'(x) = \arctan^3(x)$

(5) $$h(x) = \frac{1}{1-\underbrace{(x-x^2+2x^3)}_{y}} = 1+y+y^2+y^3+O(y^4) =$$

$$= 1+x-x^2+2x^3+(x-x^2+2x^3)^2+(x-x^2+2x^3)^3+O(x^4)$$

$$= 1+x-x^2+2x^3+x^2-2x^3+x^3+O(x^4)$$

$$= 1+x+x^3+O(x^4)$$

(6) $$\cos(x)+\frac{1}{2}x\sin(x)-1 = \cancel{1}-\cancel{\frac{x^2}{2}}+\frac{x^4}{4!}+\frac{x}{2}\left(\cancel{x}-\frac{x^3}{3!}\right)+O(x^6)$$

$$= -\frac{1}{24}x^4+O(x^6)$$

$$e^{x^2} = 1+x^2+\frac{x^4}{2}+O(x^5)$$

$$\lim_{x\to 0}\frac{\cos(x)+\frac{x}{2}\sin(x)-1}{e^{x^2}-1-x^2} = \lim\frac{-\frac{1}{24}x^4+O(x^6)}{\frac{x^4}{2}+O(x^5)} = -\frac{1}{12}$$