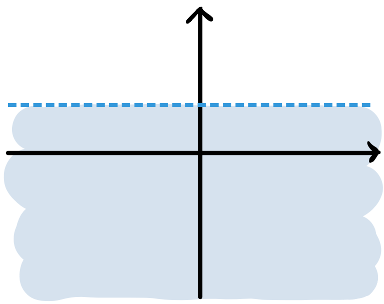
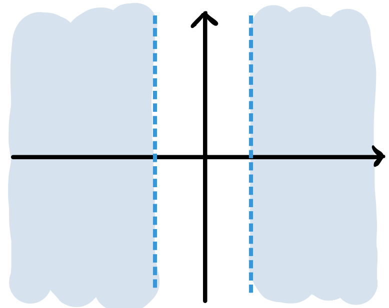
MoPC4A - CC - 17/3/2023

① a)



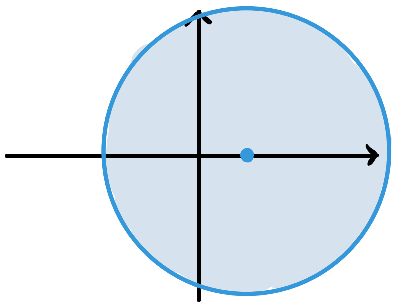
OUI

b)



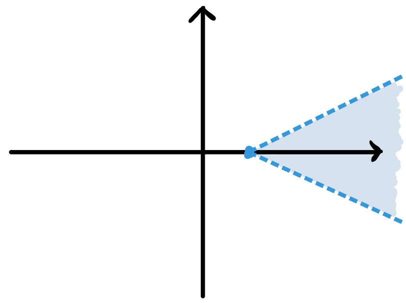
NON

c)



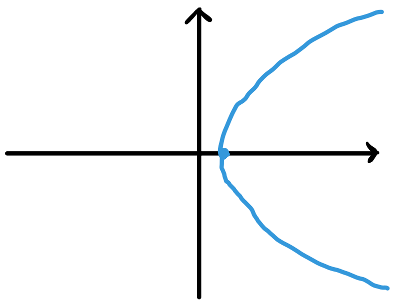
NON

d)



OUI

e)



$$x = \frac{1}{2}(1+y^2)$$

NON

② a)

- $$\frac{1+i}{1+3i} = \frac{(1+i)(1-3i)}{(1+3i)(1-3i)} = \frac{4-2i}{1+9} = \frac{2}{5} - \frac{1}{5}i$$

- $$1+i = |1+i|\, e^{i \arg(1+i)} = \sqrt{2}\, e^{i\pi/4}$$

$$(1+i)^8 = \sqrt{2}^8\, e^{i\pi/4 \cdot 8} = 2^4\, e^{2i\pi} = 16$$

- $$\mathrm{Log}(ie) = \mathrm{Log}(e\, e^{i\pi/2}) = \mathrm{Log}\, e^{1+i\pi/2} = 1 + i\frac{\pi}{2}$$

b) • $\left| -\frac{1}{2} + i\frac{\sqrt{3}}{2} \right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$-\frac{1}{2} + i\frac{\sqrt{3}}{2} = \cos\phi + i\sin\phi$

$\Rightarrow \arg\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \phi = \frac{2\pi}{3} \pmod{2\pi}$

• $\left|\exp\left(i\, e^{7\pi i/4}\right)\right| = \exp\left(\operatorname{Re} i\, e^{7\pi i/4}\right) =$

$= \exp\left(-\operatorname{Im} e^{7\pi i/4}\right) = \exp\left(-\sin\frac{7\pi}{4}\right)$

$= \exp\left(\frac{1}{\sqrt{2}}\right)$

$\arg\left(\exp\left(i\, e^{7\pi i/4}\right)\right) = \operatorname{Im}\left(i\, e^{7\pi i/4}\right) =$

$= \operatorname{Re}\left(e^{7\pi i/4}\right) = \cos\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}}$

③ $f = u + iv \qquad u = x^3 - 3xy^2 \qquad v = 3x^2y - y^3$

CR : $u_x = 3x^2 - 3y^2 = v_y$

$v_x = 6xy = -u_y$

④ $C = [0, 1+i]$ donc $z(t) = (1+i)t \quad t \in [0,1]$

$z(t) = x(t) + iy(t) \Rightarrow x(t) = t,\ y(t) = t$

$\int_C (x - y + ix^2)\, dz = \int_0^1 i t^2 (1+i)\, dt = (i-1) \left.\frac{t^3}{3}\right|_0^1 =$

$= \frac{1}{3}(i-1)$

⑤ $\displaystyle\int_{C'} \frac{1}{z^2-1}\,dz = \int_{C'} \frac{f(z)}{z+1}\,dz$ où $f(z) = \dfrac{1}{z-1}$

$f(z)$ holom. dans $D_f = \mathbb{C} \setminus \{1\}$ et $D(-\frac{1}{2}, 1) \subset D_f$

Par le th. de Cauchy : $\displaystyle\frac{1}{2\pi i}\int_{C'} \frac{f(z)}{z+1}\,dz = f(-1) = -\frac{1}{2}$

⑥ $a_n = n^2$ donc $R = \displaystyle\lim_{n \to +\infty} \frac{n^2}{(n+1)^2} = 1$

$\Rightarrow$ Le domaine de convergence est : $D(1, 1)$